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Numerical solution of non-dimensional governing equations of a flat plate solar air collector with and without storage bed

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Abstract

This paper represents a numerical solution for the flow and the temperature inside a flat plate solar collector. The analysis is applied to two cases. The first case is a flat plate solar collector without a storage bed while the second case is a flat plate solar collector with a storage bed. An explicit finite difference method provided good results for the temperature distribution inside the collector and the storage bed. The results show that the air velocity at the outlet of the collector has increased by 38% of the air velocity at the entrance. The effect of using the storage bed is to reduce the outlet air maximum temperature by about 26% of the temperature reached without using the storage bed. There is a 2- hour lag between the maximum temperature obtained for the case without the storage bed and with the storage bed.

1. Introduction

Flat plate solar collectors are a simple cheap way to convert solar radiation into heat energy that can be used in many applications. For example it can be used in the task of heating and drying, water heating domestically and industrially and many other applications. Solar energy is one of the energy sources associated with time where they vary from hour to hour, day to day and also from year to year. It is also associated with locations on the earth. This made researchers interested in studying this kind of energy in all over the world [1, 2].

One of the main problems associated with flat plate solar collectors is how to find a cheap effective way to store energy to be used in low solar radiation times or in times of increased energy demands for example, the period of the night. The choice of storage method depends on the nature of the applications [1].

In this paper a comparison between flat plate solar collector with storage and without storage will be conducted. Special attention will be given to the analysis of the fluid motion as well as the heat transfer inside flat solar collectors using the explicit finite difference method. The finite difference method is used due to the difficulties of solving a complete version of the governing equations mathematically. The final target of this paper is to study the possibility of storing solar energy obtained during the day in a period of solar radiation during the night.

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2. Problem Description

In this paper the solar collector has been simplified by the model shown in Figure 1. Although there has been an important research which focuses on modifying and devolvement of this type of collectors, simple flat plate air collectors are still used in many low temperature applications[1, 3].The collector consists of two parallel plates. The upper plate is a sheet of glass with high transmissivity in the range of the solar spectrum to allow access of the solar rays. The solar rays eventually fall on the bottom board. The bottom board is a black painted flat plate with high absorptivity of solar radiation. The thermal energy is extracted from the collector by receiving the solar radiation from the sun directly as implemented rays through the glass across the air gap to fall on the board which absorbs these rays leading to a rise in the temperature of the metal. When the air passes over this board the heat transfers from the metal to the air. The amount of heat transmitted to the air depends on several factors such as the temperature of the outside air, as well as the intensity of solar radiation.

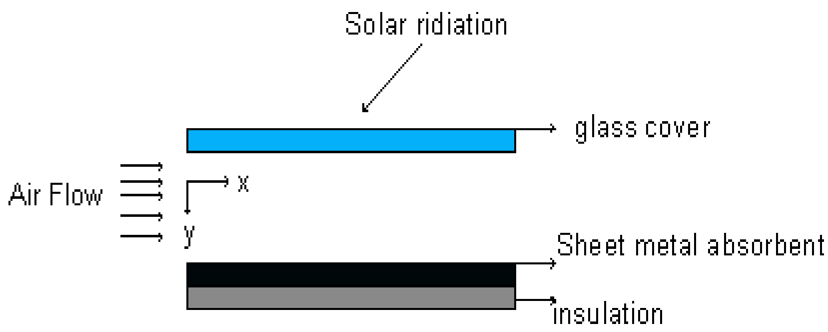

Figure 1 : Flat plate solar collector without storage bed.

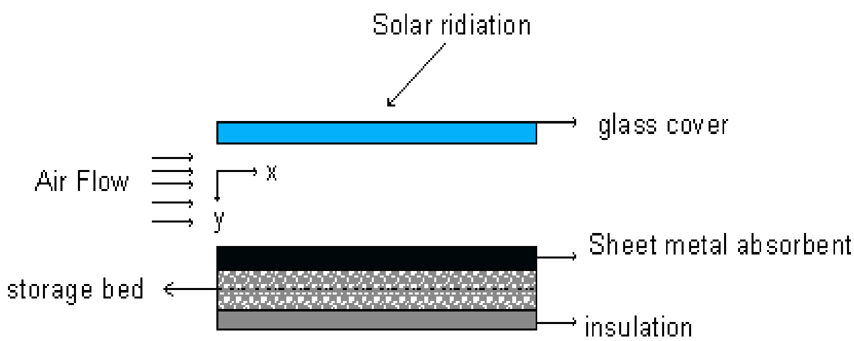

Figure 2 : Flat plate solar collector with a storage bed.

Figure 2 shows the same collector with the solar thermal storage tank is added[4]. The storage is used to absorb the thermal energy from the metal sheet in the day to be returned to the air passing through the collector in the night period. The length and width of the thermal reservoir are the same as the length and width of collector[5], while its depth is Hs.

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3. Governing equations and boundary conditions

The governing equations required to obtain both the velocity and the temperature distribution inside the collector and the storage consist of three basic equations; the continuity equation, the momentum equation and the energy equation. To obtain these equations in the form described in equations 1, 2, 3 and 4 the following assumptions were imposed. The flow of air is laminar, uniform and developed in the collector and that the properties of the air along the period of operation are fixed and that air is incompressible [6].

Continuity equation

$$\frac{\partial u}{\partial \mathrm{x}}+\frac{\partial v}{\partial \mathrm{y}}=0 \qquad 1$$

Momentum equation

$$u\frac{\partial u}{\partial \mathrm{x}}+v\frac{\partial u}{\partial \mathrm{y}}=-\frac{1}{\rho}.\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \qquad 2$$

Energy equation

$$u\frac{\partial T}{\partial \mathrm{x}}+v\frac{\partial T}{\partial \mathrm{y}}=\frac{\nu}{Pr}.\frac{\partial^2 T}{\partial y^2} \qquad 3$$

Energy equation in the storage

$$\rho_s C_s\frac{\partial T}{\partial \mathrm{t}}=K_s\frac{\partial^2 T}{\partial y^2} \qquad 4$$

The above equations have been non-dimensionalized using the following variables [5];

$$U=\frac{u}{u_m},V=v.\frac{Re_H}{u_m},X=\frac{x}{H.Re_H},Y=\frac{y}{H},P=\frac{p-p_o}{\rho.u_m^2}$$
$$\theta=\frac{(T-T_o).K_a}{q_{avr}.H},\tau=\frac{t\alpha_s}{H_s^2},Lc=\frac{L}{HRe}$$

By substituting the these dimensionless variables in equations 1 to 4, we get the following non-dimensional equations;

The continuity equation

$$\frac{\partial U}{\partial \mathrm{X}}+\frac{\partial V}{\partial \mathrm{Y}}=0 \qquad 5$$

The momentum equation

$$U\frac{\partial \mathrm{U}}{\partial \mathrm{X}}+V\frac{\partial U}{\partial \mathrm{Y}}=-\frac{\partial P}{\partial x}+\frac{\partial^2 U}{\partial Y^2} \qquad 6$$

The energy equation

$$U\frac{\partial \theta}{\partial \mathrm{x}}+V\frac{\partial \theta}{\partial \mathrm{y}}=\frac{1}{Pr}.\frac{\partial^2 \theta}{\partial Y^2} \qquad 7$$

Energy equation in the storage

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$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial Y^2} \qquad 8$$

In order for the system of equations (5 to 8) to be solved, the boundary conditions for the velocity and temperature need to be defined as follows:

- At the glass
    - The flow velocity is equal to zero in both vertical and horizontal directions.

$$U = V = 0 \; @ \; Y = 0 \qquad 9$$

    - The energy transmitted to the glass = energy lost by convection to outside air

$$K_a \frac{\partial T}{\partial y} = h_o(T_g - T_a) \quad @Y = 0 \qquad 10$$

By substituting the non-dimensional variables in the above equation we get

$$\frac{\partial \theta}{\partial \mathrm{Y}} = Nu_o(\theta_g - \theta_a) \quad @Y = 0 \qquad 11$$

- At the plate
    - The flow velocity is also equal to zero in both directions

$$U = V = 0 \; @ \; Y = 1 \qquad 12$$

    - The absorbed energy by radiation is equal to energy transmitted by conduction to the air plus the energy transmitted to the storage

$$\alpha q_i = K_s \frac{\partial T}{\partial y} + K_a \frac{\partial T}{\partial y} \quad @ \; Y = 1 \qquad 13$$

Which can be written in the non-dimensional form as follows.

$$\frac{\partial \theta}{\partial Y} = \alpha R_i - (\frac{K_s}{K_a})(\frac{H}{H_s}) \frac{\partial \theta}{\partial Y} \quad @ \; Y = 1 \qquad 14$$

- At the lower surface of the storage
    - By assuming the lower surface of the storage tank is complete insulated

$$\frac{\partial \theta}{\partial Y} = 0 \quad @ \; Y = 1 + \frac{H_s}{H} \qquad 15$$

- The initial conditions for the temperature inside the storage bed.

$$\theta = 0 \quad @ \; \tau = 0 \qquad 16$$

4. Solution procedure

There are several numerical methods that can be used to solve the system of partial differential equations derived in the last section. One of these methods is the explicit finite difference method [7] which has been used in this paper to solve equations 5 to 8 with the boundary conditions defined in equations 9,11,12,14,15 and16.

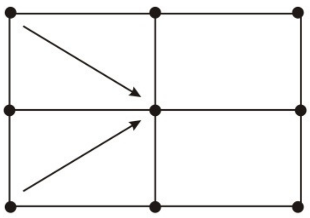

Figure 3 : Explicit finite difference scheme.

The continuity equation (equation 6) can be converted to finite difference form as follows:

$$\mathrm{U_{i,j}}\frac{\mathrm{U_{i,j}-U_{i-1,j}}}{\Delta \mathrm{x}}+\mathrm{V_{i-1,j}}\frac{\mathrm{U}_{i-1,j+1}-\mathrm{U_{i-1,j-1}}}{2\,\Delta Y}=\frac{-\,\partial \mathrm{P}}{\partial \mathrm{X}}+\frac{\mathrm{U_{i-1,J+1}+U_{i-1,j-1}-2U_{i-1,j}}}{\Delta Y^2} \qquad 17$$

This can be rearranged to find the velocity as follows:

$$\begin{aligned}\mathrm{U_{i,j}}=\mathrm{U_{i-1,j}}&-\left[\frac{\Delta X}{\mathrm{U_{i-1,j}}}\right]\frac{dP}{dX}\\&+\left[\frac{\Delta X}{\mathrm{U_{i-1,j}}}\right]\left(\frac{(-\mathrm{V_{i-1,j}})(\mathrm{U}_{i,j+1}-\mathrm{U_{i-1,j-1}})}{2\,\Delta Y}\right.\\&\left.+\frac{(\mathrm{U_{i-1,j+1}+U_{i-1,j-1}-2U_{i-1,j}})}{\Delta Y^2}\right)\end{aligned} \qquad 18$$

Also equation 5 can be written as

$$V_{i,j}=V_{i,j-1}-\left[\frac{\Delta Y}{2\Delta X}\right]\left[U_{i,j}-U_{i-1,j}+U_{i,j-1}-U_{i-1,j-1}\right] \qquad 19$$

By applying the finite difference in equation 7 the dimensionless temperature at node i and j can be expressed in the following form;

$$\begin{aligned}\theta_{\mathrm{i,j}}=\theta_{\mathrm{i-1,j}}&+\left(\frac{1}{\mathrm{Pr}}\right)\left(\frac{\Delta X}{U_{i-1,j}}\right)\left[\frac{\theta_{\mathrm{i-1,j+1}}+\theta_{\mathrm{i-1,j-1}}-2\theta_{\mathrm{i-1,j}}}{\Delta Y^2}\right]\\&-\left(\frac{\Delta X.\mathrm{V_{i-1,j}}}{U_{i-1,j}}\right)\left[\frac{\theta_{\mathrm{i-1,j+1}}-\theta_{\mathrm{i-1,j-1}}}{2\,\Delta Y}\right]\end{aligned} \qquad 20$$

Finally by converting equation 8 into the finite difference form the temperature inside the storage tank can be expressed as

$$\left(\theta_{i,j}\right)_{New}=\left(\theta_{i,j}\right)_{old}+\Delta\tau\left[\left[\frac{\theta_{\mathrm{i,j+1}}+\theta_{\mathrm{i,j-1}}-2\theta_{\mathrm{i,j}}}{\Delta Y_{\circ}{}^{2}}\right]\left[\frac{H}{H_{\circ}}\right]^{2}\right] \qquad 21$$

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5. Results and Discussion

This section shows the results obtained from a computer program, which is written especially to solve the dimensionless governing equations using finite difference method (equations 18 to 21). An explicit finite difference method is implemented with steps as shown in the previous section (Solution procedure). The figures shown in this section has been drawn using the dimensionless numbers defined in section 3. In all figures in this section Y = 0 represents the upper plate of the collector (the glass plate), Y = 1 represents the absorber plate and Y=2 represents the lower end of the storage bed which is insulated from the outside surface.

Figure 4 shows the velocity profile of the air in the horizontal direction (x-direction) at different sections along the collector. It applies for both the solar collector without storage bed (Figure 1) and with storage bed (Figure 2). The figure shows that the flow is developing flow as the velocity increasing along the x-direction. The maximum velocity occurs at the midpoint between the glass and the absorber plate. The figure also shows that the air velocity at the outlet of the collector has increased by 38 percentage of the air velocity at the entrance.

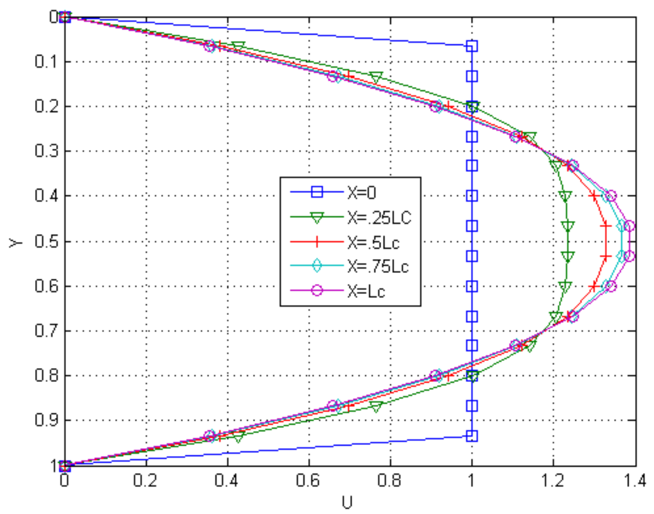

Figure 4: the velocity profile at different sections of the collector.

Figure 5 shows the temperature distribution at the air outlet of the collector without storage bed for different times of the day while Figure 6 shows the temperature distribution at the outlet of the collector with the storage bed. The maximum dimensionless temperature (θ) reached in both cases were on the absorber plate which occur after 5-hours without the storage and after 7-hours with the storage. However, the effect of using the storage bed is to reduce the outlet air temperature to about 26% of the temperature reached without using the storage bed. In both cases, with and without storage bed, the upper layers of air temperature were not affected by the plate temperature, i.e. the temperature is equal to the ambient temperature in the upper layers of the air, which may affects the efficiency of the solar collector during the day. Also it is clear that the storage bed did not reach the maximum steady temperature.

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After the end of the period of the daylight and the stop of solar radiation, the solar collector primitive stage of discharging heat from the storage bed to the air to take advantage of it in the night. It is clear that the temperature drops dramatically, immediately after the solar radiation has stopped (see **Figure 1**). Then the heat continues to escape from the tank to heat the air until it reaches the lowest temperature values after a full day (24-hours). During the night period, the air temperature inside the collector without the storage bed is supposed to be equal to the ambient temperature. In the case of using the storage bed the temperature of the exit air still greater than the temperature of the ambient as shown in Figure 7. That is due to the heat transfer from the storage bed to the air passing through the collector.

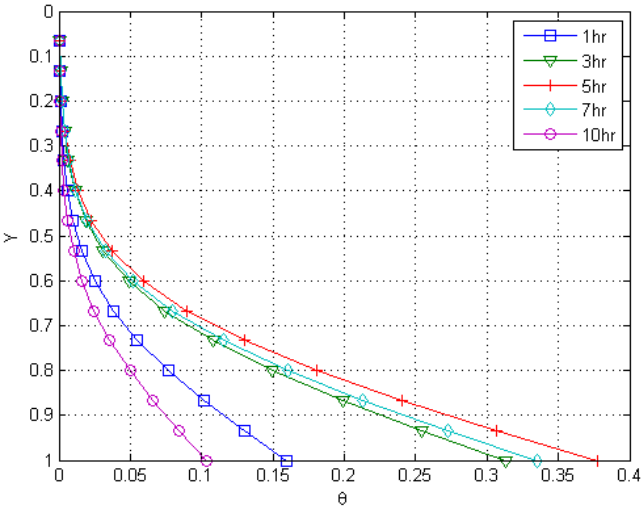

Figure 5 : Temperature distribution at different times during the daylight period for the collector without a storage bed.

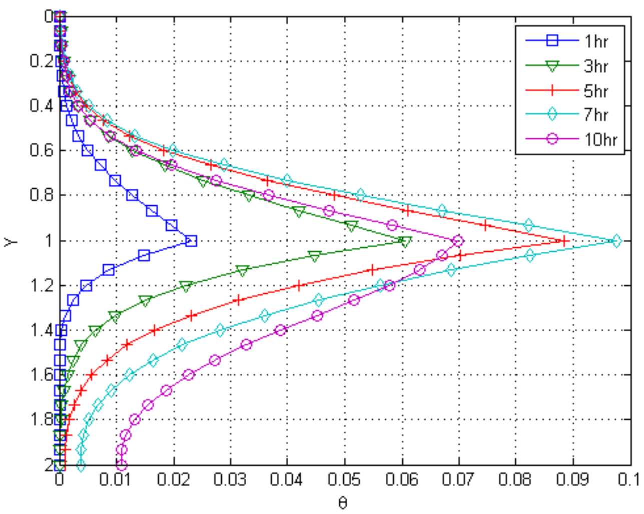

Figure 6: Temperature distribution at different times during the daylight period for the collector with a storage bed.

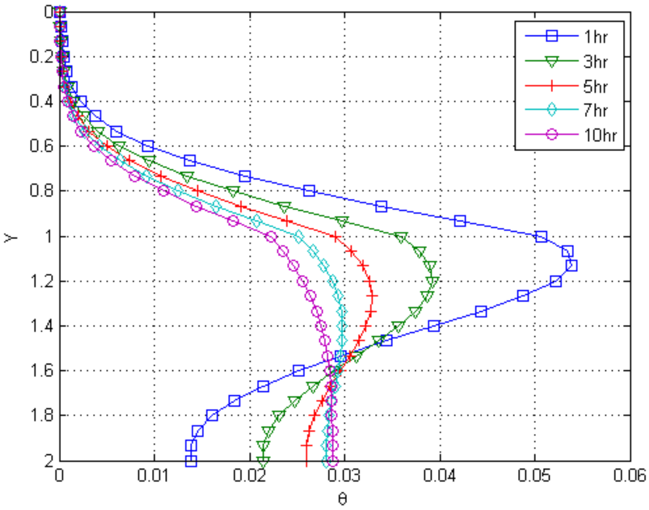

Figure 7: Temperature distribution at different times during the night period for the collector with storage bed.

Finally, Figure 8 and Figure 9 show the temperature distribution in the solar collector without the storage bed and in the solar collector with the storage bed respectively. The temperature profile for both cases show a gradient increase in the temperature with the maximum increase occur adjacent to the absorber plate as shown in previous figures. Another trend can be drawn from Figure 8 and Figure 9 is that there is a 2- hours delay in the peak temperature of the air during the day when using the storage bed. As shown in the figures the peak occurs after 5- hours for the collector without the storage bed and after 7- hours for the collector with the storage bed.

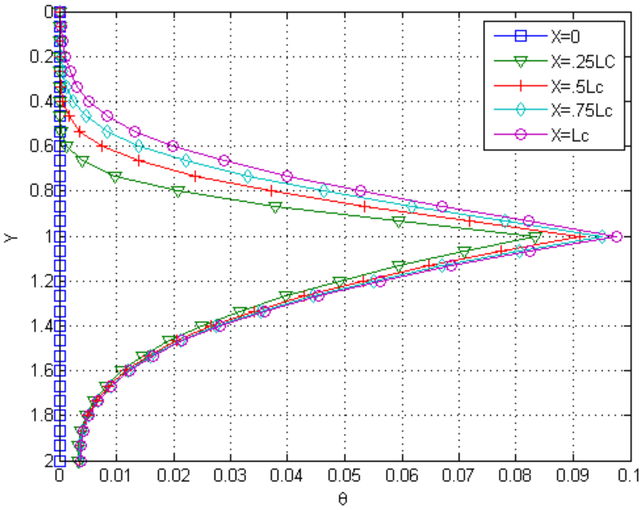

Figure 8: Maximum temperature distribution (7hrs) at different sections of the collector with storage bed.

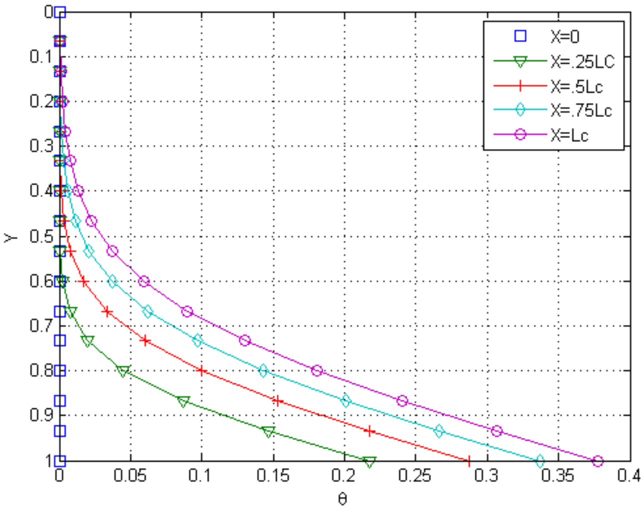

Figure 9: Maximum temperature distribution (5hrs) at different sections of the collector without storage bed.

6. Conclusions

In this paper, numerical analysis using the explicit finite difference method is used in order to solve dimensionless governing equations for air solar collector without and with a storage bed. The results showed that the air velocity at the outlet of the collector has increased by 38% of the air velocity at the entrance. The effect of using the storage bed is to reduce the maximum outlet air temperature to about 26% of the temperature reached without using the storage bed. There is a 2-hours lag between the maximum temperature obtained for the case without the storage bed and with the storage bed. More work is still needed to improve the solution method to take into account some important parameters, for example the change in the ambient temperature.

List of symbols

Symbol	*Definition*
C	specific heat
H	space between glass and absorber in the collector
H_s	storage bed height
K_a	Thermal conductivity of the air
K_s	Thermal conductivity of the storage bed material
L_C	collector length
P	dimensionless pressure
p	pressure
P_0	ambient pressure
Pr	Prandtl number
q_{avr}	average solar radiation
qi	hourly solar radiation
Re_H	Reynolds number based on H
R_i	dimensionless hourly solar radiation
T	temperature
T_0	ambient temperature
t	time
U	dimensionless velocity component in x-direction
u	velocity component in x-direction
u_m	average velocity in x-direction
V	dimensionless velocity component in v-direction
v	velocity component in y-direction
x	horizontal direction
y	vertical direction

Greek Symbols

α_s	absorptivity
θ	dimensionless temperature
ρ	air density
ρ_s	storage material density
τ	dimensionless time

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